Precalc Warm Up # 8-4

1. Is $\cos x + \sin x = 1$ true for all x? Can you find any x for which it is true?

2. Is $\cos^2 x + \sin^2 x = 1$ true for all x? Prove it.

3. Simplify: $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x}$

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In Exercises 15–20, match the trigonometric expression with one of the following.

(a) -1 (b) $\cos x$ (c) $\cot x$
(d) 1 (e) $-\tan x$ (f) $\sin x$

17. $\tan^2 x - \sec^2 x$

In Exercises 21–26, match the trigonometric expression with one of the following.

(a) $\csc x$ (b) $\tan x$ (c) $\sin^2 x$
(d) $\sin x \tan x$ (e) $\sec^2 x$ (f) $\sec^2 x + \tan^2 x$

21. $\sin x \sec x$

25. $\sec^4 x - \tan^4 x$

$(\sec^2 x + \tan^2 x)(\sec^2 x - \tan^2 x)$

In Exercises 27–40, use the fundamental identities to simplify the given expression.

29. $\cos\beta\tan\beta$

33. $\sec^2 x(1-\sin^2 x)$

39. $\dfrac{\cos^2 y}{1-\sin y}$

$$\frac{\cos^2 y}{1-\sin y}\cdot\frac{1+\sin y}{1+\sin y}$$

$$\frac{\quad}{1-\sin^2 y}$$

$\vdots$

In Exercises 41–48, factor each expression and use the fundamental identities to simplify the result.

43. $\sin^2 x\sec^2 x-\sin^2 x$

$\sin^2 x(\sec^2 x-1)$

47. $\sin^4 x-\cos^4 x$

$(\sin^2 x-\cos^2 x)$

$1-\cos^2 x-\cos^2 x$

$1-2\cos^2 x$

In Exercises 49–52, perform the multiplication and use the fundamental identities to simplify the result.

51. $(\sec x+1)(\sec x-1)$

In Exercises 53–56, perform the addition and use fundamental identities to simplify the result.

55. $\frac{(1-\sin x)}{(1-\sin x)} \cdot \frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$

$\frac{\cos x(1-\sin x)}{1-\sin^2 x} + \frac{1+\sin x}{\cos x}$

$\frac{\cancel{\cos x}(1-\sin x)}{\cos^2 x} + \frac{1+\sin x}{\cos x}$

$\frac{1-\sin x + 1 + \sin x}{\cos x}$

$\frac{2}{\cos x}$

$\boxed{2 \sec x}$

In Exercises 57–60, rewrite the given expression so that it is *not* in fractional form.

59. $\frac{3}{\sec x - \tan x}$

In Exercises 61–70, use the given trigonometric substitution to write the algebraic expression as a trigonometric expression involving θ, where $0 < \theta < \pi/2$.

63. $\sqrt{x^2 - 9}, \quad x = 3 \sec \theta$

69. $\sqrt{(9 + x^2)^3}, \quad x = 3 \tan \theta$

$\left(\sqrt{9+(3\tan\theta)^2}\right)^3$

$\left(\sqrt{9+9\tan^2\theta}\right)^3$

$\left(\sqrt{9(1+\tan^2\theta)}\right)^3$

$\left(3\sqrt{\sec^2\theta}\right)^3$

$\boxed{27 \sec^3\theta}$

$\sqrt{x^3} = \left(\sqrt{x}\right)^3$

73. $\ln|\cos\theta| - \ln|\sin\theta|$

$\ln\left|\frac{\cos\theta}{\sin\theta}\right|$

$\ln|\cot\theta|$

In Exercises 75–78, determine whether the statement is true or false, and give a reason for your answer.

75. $\frac{\sin k\theta}{\cos k\theta} = \tan\theta$, k is constant false.

$\frac{\sin k\theta}{\cos k\theta} = \tan k\theta$, not $\tan\theta$

Identities are true for all x, whereas equations are true for some x or none at all. Today, we will prove identities. There are an infinite number of them, with more being created every day!

Guidelines for verifying...

1. Start with one side of the equation and try to turn it into the other side. Usually easiest to start on the more complicated side and simplify it.

2. Look for opportunities to factor, add fractions, multiply numerator or denominator of a fraction by something that will create squares to take advantage of one of the pythagorean identities, or to create a monomial in the denominator.

3. Look for opportunities to use the FUNDAMENTAL identities, all the while keeping in mind what types of terms you are going for. Sines and Cosines pair up well (why?) as do Sec and Tan, and Csc and Cot.

4. DO NOT JUST SIT AND STARE AT IT !!! TRY SOMETHING !!!!

Verify:

1. $$\frac{\sec^2 x-1}{\sec^2 x} = \sin^2 x$$

$\sec^2 x = 1 + \tan^2 x$

$\sec^2 x - 1 = \tan^2 x$

$$\tan^2 x \cdot \frac{1}{\sec^2 x}$$

$$\frac{\sin^2 x}{\cancel{\cos^2 x}} \cdot \frac{\cancel{\cos^2 x}}{1}$$

$$\sin^2 x$$

Verify:

2. $$\frac{(1+\sin a)}{(1+\sin a)}\frac{1}{(1-\sin a)} + \frac{1}{(1+\sin a)}\frac{(1-\sin a)}{(1-\sin a)} = 2\sec^2 a$$

$$\frac{1+\sin a}{1-\sin^2 a} + \frac{1-\sin a}{1-\sin^2 a} = 2\sec^2 a$$

$$\frac{1+\sin a+1-\sin a}{1-\sin^2 a} = 2\sec^2 a$$

$$\frac{2}{\cos^2 a} = 2\sec^2 a$$

$$2\sec^2 a = 2\sec^2 a \checkmark$$

Verify:

3. $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

$$\sec^2 x(\cos^2 x - 1) = -\tan^2 x$$

$$-\sec^2 x(1 - \cos^2 x) = -\tan^2 x$$

$$-\sec^2 x(\sin^2 x) = -\tan^2 x$$

$$-\frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} = -\tan^2 x$$

$$-\tan^2 x = -\tan^2 x \quad \checkmark$$

Verify:

4. $\tan x + \cot x = \sec x \cdot \csc x$

$$\frac{\tan^2 x}{\tan x} + \frac{1}{\tan x} = \sec x \csc x$$

$$\frac{\tan^2 x + 1}{\tan x} =$$

$$\frac{\sec^2 x}{\tan x} =$$

$$\sec x \cdot \frac{\sec x}{\tan x} =$$

$$\sec x \cdot \frac{\frac{1}{\cancel{\cos x}}}{\frac{\sin x}{\cancel{\cos x}}} =$$

$$\sec x \cdot \frac{1}{\sin x} =$$

$$\sec x \csc x = \sec x \csc x$$

Verify:

5. $$\sec y + \tan y = \frac{\cos y}{1 - \sin y}$$

Verify:

5. $$\sec y + \tan y = \frac{\cos y}{1 - \sin y} \cdot \frac{(1 + \sin y)}{(1 + \sin y)}$$

$$= \frac{\cos y(1 + \sin y)}{1 - \sin^2 y}$$

$$= \frac{\cancel{\cos y}(1 + \sin y)}{\cos^{\cancel{2}} y}$$

$$= \frac{1}{\cos y} + \frac{\sin y}{\cos y}$$

$$\sec y + \tan y = \sec y + \tan y$$

Verify: Be careful not to make it more complicated than you have to!

5. $\frac{(1-\sin y)}{(1-\sin y)}\frac{(\sec y+\tan y)}{1}=\frac{\cos y}{1-\sin y}$

$$\frac{\sec y+\tan y-\sin y\sec y-\sin y\tan y}{1-\sin y}$$

$$\frac{\sec y+\cancel{\frac{\sin y}{\cos y}}-\cancel{\frac{\sin y}{\cos y}}-\sin y\frac{\sin y}{\cos y}}{1-\sin y}$$

$$\frac{\frac{1}{\cos y}-\frac{\sin^2 y}{\cos y}}{1-\sin y}$$

$$\frac{\frac{1-\sin^2 y}{\cos y}}{1-\sin y}$$

$$\frac{\frac{\cos^{\cancel{2}} y}{\cancel{\cos y}}}{1-\sin y}$$

$$\frac{\cos y}{1-\sin y}=\frac{\cos y}{1-\sin y}\ \checkmark$$

HW: PC book

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